Heating of induction bearings

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In this paper, thermal effects in induction bearings are considered. The governing equations for a model are presented and discussed. Results of the analysis show relationships between rotating speed, frequency, and temperature of the bearing.

Keywords: heat transfer, electromagnetic field, induction bearing

Introduction

The development of possibilities of machines depends on the production of bearings that can work at high rotating speeds. A modern type of bearing that is used in mechanical industry is an induction heating bearings. This bearing has several advantages. Induction bearings do not need a lubricating substance. There is no mechanical contact between working parts of the bearing. It works silently with little friction. The lifetime of this bearing is much longer than that of a conventional one.

Analytical methods for the analysis of thermoelectric problems in induction bearings are not convenient. For the model presented, we use the finite element method to carry out calculations. The heating of induction bearings should be carefully examined, when the gap between the working parts is small because temperature increases can cause the bearing to seize up.

The theory of thermal and thermomechanical effects in other types of electric equipment is discussed in Refs. 1 and 2.

Thermoelectric problems in induction bearing

Heating of induction bearings can be considered as a two-dimensional problem. Therefore in this paper we will analyze two-dimensional heat transfer and electromagnetic field equations.

If heat is generated in a solid at a rate per unit time, then the temperature T of this solid must satisfy the following

$$\frac{1}{\partial x}\left(\lambda_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \frac{\partial T}{\partial y}\right) + p = 0 \tag{1}$$

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where A, and A, are thermal conductivities and (x, y) is a coordinate system.

The boundary condition for heat transfer at the surface $\partial\Omega$ into the surrounding medium with temperature T_s according to Newton's equation is

$$\left[\lambda \frac{\partial T}{\partial n} + h(\hat{T} - T_s)\right]\Bigg|_{\partial\Omega} = 0 \tag{2}$$

where $\partial/\partial n$ is the derivative in the normal external direction to $\partial\Omega$, h is the heat transfer coefficient, and $\hat{T}$ is the temperature on the boundary $\partial\Omega$.

The rate of heat generated by electric current has the form[5]

$$p = \gamma|\mathbf{E}|^2 \tag{3}$$

where E is the complex root mean square value of an electric field, $|z|$ is the modulus of the complex number z, and γ is the electrical conductivity. Equation (3) can be transformed to the following form, which shows the heat generation per unit length:

$$p = \frac{\omega}{\mu} k^2 |A|^2 \tag{4}$$

where ω is the pulsation, μ is the permeability, $k^2 = \omega\mu\gamma$, and A is the vector potential.

Consider a conductor in motion. When the vector potential has only the component along the conductor, then it satisfies the following equation?

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A}{\partial y}\right) = -J_w + \gamma\left(V_x \frac{\partial A}{\partial x} + V_y \frac{\partial A}{\partial y}\right) + j\gamma\omega A \tag{5}$$

where V_x and V_y denote components of the velocity, J_w is the linear density of the current, and $j = \sqrt{-1}$.

The boundary conditions for (5) are considered as the Dirichlet problem

$$A(P) = f(P) \qquad P \in \partial\Omega \tag{6}$$

$$\left.\frac{\partial A}{\partial n}\right|_P = g(P) \qquad P \in \partial\Omega \tag{7}$$

Heat flow equations

The problems of the finite element solution of the heat flow equation are analyzed in many textbooks (see Refs. 4 and 7). To show similar constructions in matrices appearing in heat flow and electromagnetic field equations, we recall a brief description of this problem. Readers who are interested in other theoretical aspects can acquaint themselves with the details by referring to the work cited.

The finite element equation of heat flow considering a linear triangular element can be expressed as

$$\boldsymbol{K}\tilde{\mathbf{T}} + \mathbf{Q} = 0 \tag{8}$$

where $\tilde{\mathbf{T}} = \{T_i, T_j, T_k\}$ denotes the column vector of nodal temperatures for finite element.

The matrix K and the vector Q have the form

$$\boldsymbol{K} = \boldsymbol{K}^k + \boldsymbol{K}^c \tag{9}$$

$$\boldsymbol{K}^k = \int_{\Omega^e} \boldsymbol{B}^T \boldsymbol{\lambda} \boldsymbol{B}\, d\Omega \tag{10}$$

$$\boldsymbol{\lambda} = \begin{bmatrix} \lambda_x & 0 \\ 0 & \lambda_y \end{bmatrix} \tag{11}$$

$$\boldsymbol{K}^c = \int_{\partial\Omega^e} h \mathbf{N}^{s^T} \mathbf{N}^s\, d(\partial\Omega) \tag{12}$$

$$\mathbf{Q} = \int_{\Omega_e} \mathbf{N}^T p\, d\Omega + \int_{\partial\Omega^e} \mathbf{N}^{s^T} \mathbf{N}^s h \tilde{\mathbf{T}}_s\, d(\partial\Omega) \tag{13}$$

Vectors N and $\mathbf{N}^s$ define the temperatures within the analyzed region or on its boundary as a function of the nodal point temperatures, and the vector superscript $\boldsymbol{T}$ denotes a vector transpose.

Assuming that

$$\boldsymbol{T} = \mathbf{N}\tilde{\mathbf{T}} \tag{14}$$

the shape functions vector N can be expressed in the system of coordinates (x, y) as[4]

$$\begin{aligned} N_i &= L_i(2L_i - 1) \\ N_j &= L_j(2L_j - 1) \\ N_k &= L_k(2L_k - 1) \end{aligned} \tag{15}$$

where

$$\begin{aligned} L_i &= \frac{1}{2\Delta}(a_i + b_i x + c_i y) \\ L_j &= \frac{1}{2\Delta}(a_j + b_j x + c_j y) \\ L_k &= \frac{1}{2\Delta}(a_k + b_k x + c_k y) \end{aligned} \tag{16}$$

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} \tag{17}$$

$$\begin{aligned} a_i &= x_j y_k - x_k y_j \\ b_i &= y_j - y_k \\ c_i &= x_k - x_j \end{aligned} \tag{18}$$

where x_i and y_i denote coordinates for node $\boldsymbol{i}$.

The matrix $\boldsymbol{B}$ defines the temperature gradient as a function of the nodal point temperatures[7]

$$\nabla T = \boldsymbol{B}\tilde{\mathbf{T}} \tag{19}$$

When we assume that $\lambda_x = \lambda_y = \lambda$, then we have from (10)

$$\boldsymbol{K}^k = \frac{\lambda}{4\Delta}\begin{bmatrix} b_i^2 + c_i^2 & b_i b_j + c_i c_j & b_i b_k + c_i c_k \\ b_i b_j + c_i c_j & b_j^2 + c_j^2 & b_j b_k + c_j c_k \\ b_i b_k + c_i c_k & b_j b_k + c_j c_k & b_k^2 + c_k^2 \end{bmatrix} \tag{20}$$

Electromagnetic field equations

Based on the Galerkin method we can transform (5) to the form

$$\int_\Omega \left(\operatorname{grad} A \operatorname{grad} N_1^* + \mu\gamma\left(v_x \frac{\partial A}{\partial x} + v_y \frac{\partial A}{\partial y} + j\omega_p A \right) N_1^* \right) d\Omega = \int_{\partial\Omega} \frac{\partial A}{\partial n} N_1^*\, d(\partial\Omega)$$

where N_1^* is the conjugate number with N_1.

The finite element equations system takes the form

$$\mathrm{L}\mathbf{A} + \mu\gamma(\boldsymbol{P} + j\omega_p \boldsymbol{W})\mathbf{A} = d\mu \mathbf{D}\mathbf{J}_w \tag{22}$$

where

$$\boldsymbol{L} = \frac{1}{4\Delta}\begin{bmatrix} b_i^2 + c_i^2 & b_i b_j + c_i c_j & b_i b_k + c_i c_k \\ b_i b_j + c_i c_j & b_j^2 + c_j^2 & b_j b_k + c_j c_k \\ b_i b_k + c_i c_k & b_j b_k + c_j c_k & b_k^2 + c_k^2 \end{bmatrix} \tag{23}$$

$$\boldsymbol{P} = \boldsymbol{P}_1 + \boldsymbol{P}_2 \tag{24}$$

$$\boldsymbol{P}_1 = \frac{1}{6}\begin{bmatrix} v_{x_i} b_i & v_{x_j} b_j & v_{x_k} b_k \\ v_{x_i} b_i & v_{x_j} b_j & v_{x_k} b_k \\ v_{x_i} b_i & v_{x_j} b_j & v_{x_k} b_k \end{bmatrix} \tag{25}$$

$$\boldsymbol{P}_2 = \frac{1}{6}\begin{bmatrix} v_{y_i} c_i & v_{y_j} c_j & v_{y_k} c_k \\ v_{y_i} c_i & v_{y_j} c_j & v_{y_k} c_k \\ v_{y_i} c_i & v_{y_j} c_j & v_{y_k} c_k \end{bmatrix} \tag{26}$$

$$W = \frac{\Delta}{12}\left[\begin{matrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix}\right] \qquad (27)$$

$$D = \frac{1}{6}\left[\begin{matrix} 2 & 1 \\ 1 & 2 \end{matrix}\right] \qquad (28)$$

and d is the distance between corresponding nodal points on the boundary.

It should be noted that finite element discretization will not be very good if first derivative terms are large, that is, bad for $|\mu V_x H| > 2$, where H is a typical element dimension.

Numerical examples

A scheme of an induction bearing is presented in Figure 1. For an arbitrary point $P(x_0, y_0)$ we have the following components of velocity v_x, v_y:

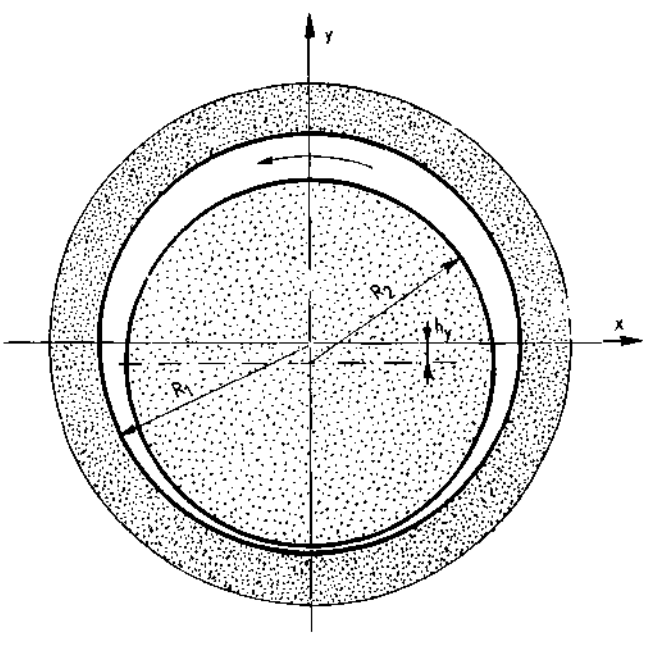

Figure 1. Scheme of an induction bearing

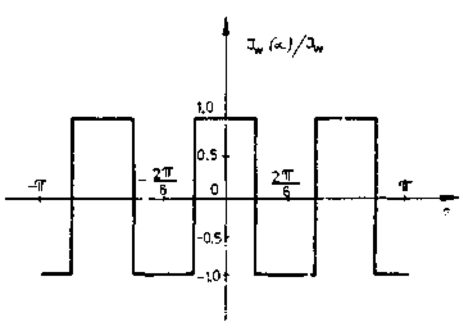

Figure 2. Assumed density of current

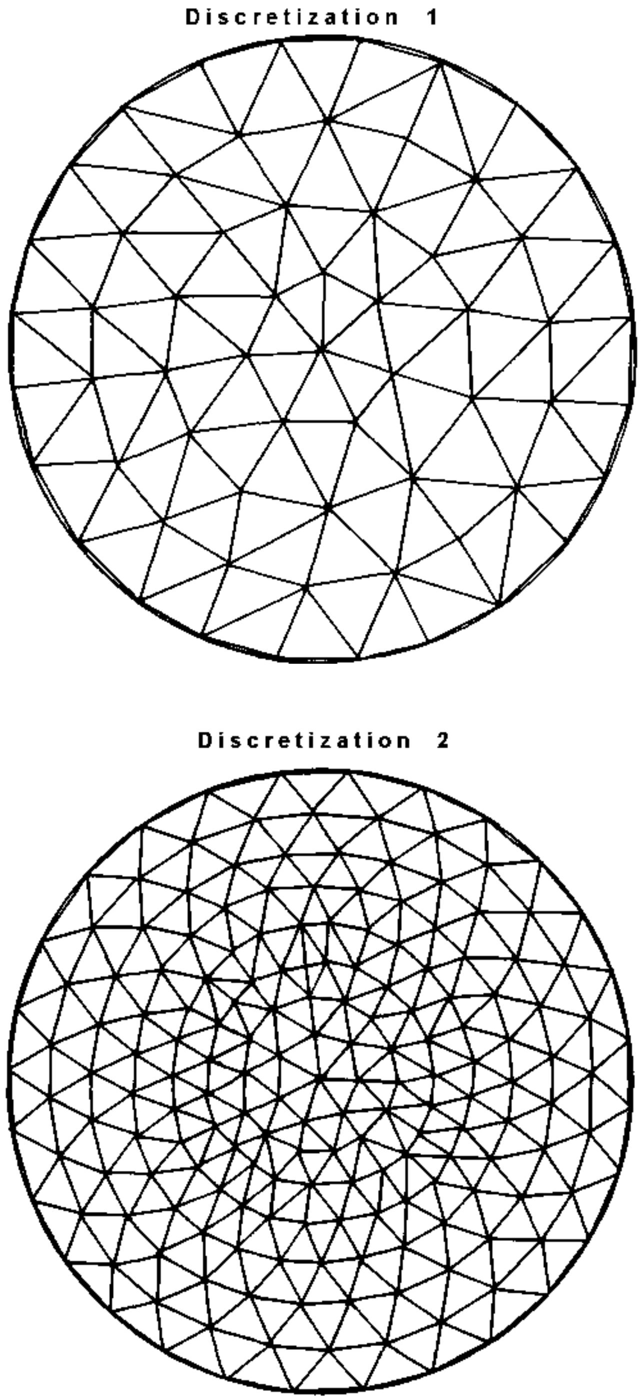

Figure 3. Finite element meshes used

$$v_x = -2\pi n r_y$$
$$v_y = 2\pi n r_x \qquad (29)$$

where

$$r_x = x_0 - h_x \qquad r_y = y_0 - h_y$$

n is the angular speed of the bearing, and h_x and h_y present displacements of the shaft center in the x- and y-directions, respectively.

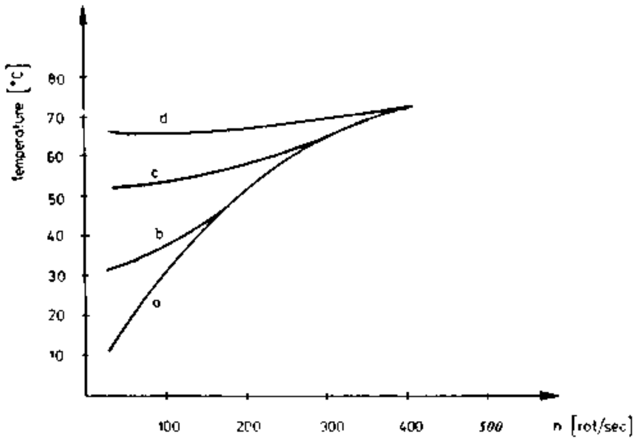

Figure 4. Relationships between temperature, rotating speed, and frequency: (a) 0 Hz, (b) 200 Hz, (c) 500 Hz, (d) 1000 Hz

The assumed density of current on the boundary is presented in *Figure 2*. Dividing the linear density of the current in Fourier series, we have

$$J_w(\alpha) = \frac{12J_w}{\pi} \sum_{i=0}^{\infty} \frac{\sin (6i+3)\frac{\pi}{6}}{6i+3} \cos mi\,\alpha \qquad (30)$$

The following parameters of the induction bearing have been assumed for the analysis: $R_1 = 0.026$ m, $R_2 = 0.020$ m, $h_y = -0.005$ m, $J_w = 90$ kA/m, $\gamma = 40 \times 10^6$ S/m, A = 384 W/mK, and $h = 0.25$ W/m². The initial temperature of the bearing is 0°C. Two finite element meshes have been used for the analysis (shown in *Figure 3*). The results of the analysis for the first discretization are presented in *Figure 4*. It was observed that results obtained for both meshes were nearly the same. The differences were in the range of 3%.

We show here only an average temperature of the bearing because in a steady state this temperature is nearly the same at all points in the bearing. Therefore it is not necessary to resort to a finite element solution for the temperature. For a temperature-independent electrical conductivity the A distribution can be solved by using FEM, and a lumped parameter technique would give the average steady-state temperature.

Final remarks

The results shown in *Figure 4* indicate that in the range of high frequencies the temperature of the bearing does not depend significantly on rotating speed. For small frequencies these variations are stronger. For a rotating speed that is higher than 400 rotations per second, the increase of temperature is small.

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